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Can primordial wormholes be induced by GUTs at the early Universe?

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abstract

Using large N, 4d anomaly induced one-loop effective action for conformally invariant matter (typical GUT multiplet) we study the possibility to induce the primordial spherically symmetric wormholes at the early Universe. The corresponding effective equations are obtained in two different coordinate frames. The numerical investigation of these equations is done for matter content corresponding to $\mathcal{N}=4$ $SU(N)$ super Yang-Mills theory. For some choice of initial conditions, the induced wormhole solution with increasing throat radius and increasing red-shift function is found.

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The wormholes are puzzling topological objects (kind of handles of topological origin) which attract much attention in General Relativity for years. It is not strange as they may be considered as bridges joining two different Universes or two separate regions of the same Universe (for an introduction, see refs.[1]). There are many speculations related with hypotetical effects which may be expected near wormholes.

Moreover, it is known that wormholes usually cannot occur as classical solutions of gravity (with matter) due to violations of energy conditions [1]. Nevertheless, one can expect that primordial wormholes may present at the very early Universe where quantum effects play an essential role (or yet in fundamental M-theory, see,for example, [2]).

Indeed as it was shown in ref.[3] (see also [4]) using quantum stress tensor for conformal scalar on spherically symmetric space and in ref.[5] using one-loop effective action in large N and s-wave approximation for minimal scalar there may exist semiclassical quantum solution corresponding to a Lorentzian wormhole connecting two asymptotically flat regions of the Universe. The corresponding spherically symmetric wormhole solution has been found numerically in both approximations [3, 5] as well as analytically [5]. That shows the principal possibility of inducing primordial wormholes at the early Universe (in its quantum regime). However above discussion has been limited strictly to scalar matter. And what happens if other types of matter are included? Hence, the very natural question is: Can primordial wormholes be induced from GUTs at the early Universe?

In the present Letter we try to answer this question for spherically symmetric wormholes. We use 4d conformal anomaly induced effective action in one-loop and large N approximation so that one can neglect quantum gravitational contributions. The effective equations for an arbitrary massless GUT containing conformal scalars, spinors and vectors are explicitly obtained for two forms of spherically symmetric background. Numerical solution of these equations is presented for $\mathcal{N}=4$ super Yang-Mills theory. It is observed that it depends on the choice of the initial conditions. The initial conditions admitting inducing of primordial wormholes are also presented.

We first derive the effective action for conformally invariant matter (for a general review of effective action in curved space, see[6]). Let us start from Einstein gravity with N_0 conformal scalars χ_i, N_1 vectors A_μ and $N_{1/2}$ Dirac spinors ψ_i

$$\begin{aligned} S &= -\frac{1}{16\pi G}\int d^4x\sqrt{-g_{(4)}}\left\{R^{(4)}-2\Lambda\right\}+\int d^4x\sqrt{-g_{(4)}}\left\{\frac{1}{2}\sum_{i=1}^{N_0}\left(g_{(4)}^{\alpha\beta}\partial_\alpha\chi_i\partial_\beta\chi_i\right.\right. \\ &\quad \left.\left.+\frac{1}{6}R^{(4)}\chi_i^2\right)-\frac{1}{4}\sum_{j=1}^{N_1}F_{j\,\mu\nu}F_j^{\mu\nu}+\sum_{k=1}^{N_{1/2}}\bar\psi_k \not{D}\psi_k\right\} . \qquad (1) \end{aligned}$$

The above matter content is typical for asymptotically free GUT at high energies [6] as interaction terms and masses at strong curvature are negligible due to asymptotic freedom. For finite GUTs or asymptotically non -free GUTs one should not include masses if consider only conformally invariant theories and interaction plays no role as anyway we consider purely gravitational background.

The convenient choice for the spherically symmetric space-time is the following:

$$ds^2=f(\phi)\left[f^{-1}(\phi)g_{\mu\nu}dx^\mu dx^\nu+r_0^2 d\Omega\right] . \qquad (2)$$

where $\mu,\nu=0,1$, $g_{\mu\nu}$ and $f(\phi)$ depend only from x^1 and r_0^2 is non-essential constant.

Let us start the calculation of matter effective action on the background (2) closely following to ref. [7](see also [9]). In the calculation of effective action, we present effective action as : $\Gamma=\Gamma_{ind}+\Gamma[1,g_{\mu\nu}^{(4)}]$ where $\Gamma_{ind}=\Gamma[f,g_{\mu\nu}^{(4)}]-\Gamma[1,g_{\mu\nu}^{(4)}]$ is conformal anomaly induced action which is quite well-known [8], $g_{\mu\nu}^{(4)}$ is metric (2) without multiplier in front of it, i.e., $g_{\mu\nu}^{(4)}$ corresponds to

$$ds^2=\left[\tilde g_{\mu\nu}dx^\mu dx^\nu+r_0^2 d\Omega\right]\ ,\quad \tilde g_{\mu\nu}\equiv f^{-1}(\phi)g_{\mu\nu}\ . \tag{3}$$

The conformal anomaly for above matter is well-known

$$T=b\left(F+\frac{2}{3}\Box R\right)+b'R_{GB}+b''\Box R \tag{4}$$

where $b=\frac{(N_0+6N_{1/2}+12N_1)}{120(4\pi)^2}$, $b'=-\frac{(N_0+11N_{1/2}+62N_1)}{360(4\pi)^2}$, $b''=0$ but in principle, b'' may be changed by the finite renormalization of local counterterm in gravitational effective action, F is the square of Weyl tensor, R_{GB} is Gauss-Bonnet invariant.

Conformal anomaly induced effective action Γ_{ind} may be written as follows [8]:

$$\begin{aligned}W &= b\int d^4x\sqrt{-g}F\sigma+b'\int d^4x\sqrt{-g}\Big\{\sigma\Big[2\Box^2+4R^{\mu\nu}\nabla_\mu\nabla_\nu \\ &\quad -\frac{4}{3}R\Box+\frac{2}{3}(\nabla^\mu R)\nabla_\mu\Big]\sigma+\left(R_{GB}-\frac{2}{3}\Box R\right)\sigma\Big\} \\ &\quad -\frac{1}{12}\left(b''+\frac{2}{3}(b+b')\right)\int d^4x\sqrt{-g}\left[\{R-6\Box\sigma-6(\nabla\sigma)(\nabla\sigma)\}^2-R^2\right]\end{aligned} \tag{5}$$

where $\sigma=\frac{1}{2}\ln f(\phi)$, and σ-independent terms are dropped. All 4-dimensional quantities (curvatures, covariant derivatives) in Eq.(5) should be calculated on the metric (3). The calculation of $\Gamma[1,g_{\mu\nu}^{(4)}]$ is done in ref. [7] in leading order as follows:

$$\Gamma[1,g_{\mu\nu}^{(4)}]=\int d^4x\sqrt{-g}\left\{\left[bF+b'R_{GB}+\frac{2b}{3}\Box R\right]\ln\frac{R}{\mu^2}\right\}+\mathcal{O}(R^3) \tag{6}$$

where μ is mass-dimensional constant parameter, all the quantities are calculated on the background (3). The condition of application of above expansion is $|R|<R^2$ (curvature is nearly constant), in this case we may limit by only few first terms.

Let us solve the equations of motion obtained from the above effective Lagrangians $S+\Gamma$. In the following, we use $\tilde g_{\mu\nu}$ and σ as a set of independent variables and we write $\tilde g_{\mu\nu}$ as $g_{\mu\nu}$ if there is no confusion.

Γ_{ind} (W in Eq.(5)) is rewritten after the reduction to 2 dimensions as follows:

$$\begin{aligned}\frac{\Gamma_{ind}}{4\pi} &= \frac{br_0^2}{3}\int d^2x\sqrt{-g}\left(\left(R^{(2)}+R_\Omega\right)^2+\frac{2}{3}R_\Omega R^{(2)}+\frac{1}{3}R_\Omega^2\right)\sigma \\ &\quad +b'r_0^2\int d^2x\sqrt{-g}\Big\{\sigma\Big(2\Box^2+4R^{(2)\mu\nu}\nabla_\mu\nabla_\nu-\frac{4}{3}(R^{(2)}+R_\Omega)\Box \\ &\quad +\frac{2}{3}(\nabla^\mu R^{(2)})\nabla_\mu\Big)\sigma+\left(2R_\Omega R^{(2)}-\frac{2}{3}\Box R^{(2)}\right)\sigma\Big\} \\ &\quad -\frac{1}{12}\left\{b''+\frac{2}{3}(b+b')\right\}r_0^2\int d^2x\sqrt{-g} \\ &\quad \times\left\{\left(R^{(2)}+R_\Omega-6\Box\sigma-6\nabla^\mu\sigma\nabla_\mu\sigma\right)^2-\left(R^{(2)}+R_\Omega\right)^2\right\}\end{aligned} \tag{7}$$

Here $R_\Omega = \frac{2}{r_0^2}$ is scalar curvature of S^2 with the unit radius.

Let us derive the effective equations of motion. In the following, we work in the conformal gauge : $g_{\pm\mp} = -\frac{1}{2}\mathrm{e}^{2\rho}$, $g_{\pm\pm} = 0$ after considering the variation of the effective action $\Gamma + S$ with respect to $g_{\mu\nu}$ and σ. Note that the tensor $g_{\mu\nu}$ under consideration is the product of the original metric tensor and the σ-function $\mathrm{e}^{-2\sigma}$, the equations given by the variations over $g_{\mu\nu}$ are the combinations of the equations given by the variation over the original metric and σ-equation.

It often happens that we can drop the terms linear in σ in (7). In particular, one can redefine the corresponding source term as it is in the case of IR sector of 4D QG [11]. In the following, we only consider this case. Then the variations of $S + \Gamma_{ind} + \Gamma[1, g_{\mu\nu}^{(4)}]$ with respect to $g^{\pm\pm}$, ρ and σ are given by (see also [7])

$$
\begin{aligned}
0 \;=\;& -\frac{r_0^2}{48\pi G}\mathrm{e}^{2\rho+2\sigma}\left[(\partial_r\sigma)^2 - \partial_r^2\sigma + 2\partial_r\sigma\partial_r\rho\right] + \frac{b' r_0^2}{16}\left[-8\mathrm{e}^{2\rho}\partial_r\sigma\partial_r\left(\mathrm{e}^{-2\rho}\partial_r^2\sigma\right)\right. \\
& \left. +8\sigma\partial_r^2\sigma\partial_r^2\rho + \frac{8}{3}\mathrm{e}^{2\rho}\partial_r\sigma\partial_r\left\{R_4\sigma\right\} + \frac{32}{3}\mathrm{e}^{2\rho}\sigma\partial_r R_4\partial_r\sigma\right] \\
& -\left\{b'' + \frac{2}{3}(b+b')\right\}\frac{r_0^2}{16}\left[16\mathrm{e}^{2\rho}\partial_r\sigma\partial_r R_4 + 4(\partial_r\sigma)^2\partial_r^2\rho\right. \\
& \left. -12\mathrm{e}^{2\rho}\partial_r\sigma\partial_r\left\{\mathrm{e}^{-2\rho}\left(\partial_r^2\sigma + (\partial_r\sigma)^2\right)\right\} + 12\left(\partial_r^2\sigma + (\partial_r\sigma)^2\right)(\partial_r\sigma)^2\right] \\
& -\frac{r_0^2}{16}\left\{\frac{1}{2}\partial_r^2\rho - 2(\partial_r\rho)^2 - \frac{1}{4}\partial_r^2 + \frac{3}{2}\partial_r\rho\partial_r\right\} \\
& \times\left[\frac{16}{3}b'\left(-\sigma\partial_r^2\sigma + (\partial_r\sigma)^2\right) - \left\{b'' + \frac{2}{3}(b+b')\right\}\left(\partial_r^2\sigma + (\partial_r\sigma)^2\right)\right] \\
& +r_0^2\left[-\frac{1}{12}b\mathrm{e}^{2\rho}\partial_r\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}\partial_r R_4\right. \\
& -\left\{\frac{1}{2}\partial_r^2\rho - 2(\partial_r\rho)^2 - \frac{1}{4}\partial_r^2 + \frac{3}{2}\partial_r\rho\partial_r\right\} \\
& \times\left\{\frac{2}{3}b\partial_r^2\rho\ln\left(\frac{R_4}{\mu^2}\right) + \frac{b}{4}\partial_r^2\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}\right. \\
& \left.\left. +\left\{b\left(\frac{2}{3}\mathrm{e}^{-2\rho}(\partial_r^2\rho)^2 + \frac{2\mathrm{e}^{2\rho}}{3r_0^2} + \frac{1}{3}\partial_r^2 R_4\right) + \frac{4}{r_0^2}\left(\frac{b}{3} + b'\right)\partial_r^2\rho\right\}\frac{1}{R_4}\right\}\right] , \qquad (8)\\
R_4 \;\equiv\;& -2\mathrm{e}^{-2\rho}\partial_r^2\rho + \frac{2}{r_0^2} , \qquad (9)\\
0 \;=\;& -\frac{r_0^2}{16\pi G}\left[-\partial_r^2\mathrm{e}^{2\sigma} + \frac{4}{r_0^2}\mathrm{e}^{2\rho+2\sigma}\right] + b' r_0^2\left\{-2(\partial_r^2\sigma)^2\mathrm{e}^{-2\rho}\right. \\
& -\frac{8}{3}\mathrm{e}^{-2\rho}\partial_r^2\rho(\sigma\partial_r^2\sigma) + \frac{4}{3}\partial_r^2\left(\mathrm{e}^{-2\rho}\sigma\partial_r^2\sigma\right) \\
& \left.-\frac{1}{3}\left\{-2\left(\partial_r\sigma\right)^2\mathrm{e}^{-2\rho}\partial_r^2\rho + \partial_r^2\left(\left(\partial_r\sigma\right)^2\mathrm{e}^{-2\rho}\right)\right\}\right\} \\
& -\left\{b'' + \frac{2}{3}(b+b')\right\}r_0^2\left[\partial_r^2\left\{\mathrm{e}^{-2\rho}\left(\partial_r^2\sigma + (\partial_r\sigma)^2\right)\right\}\right. \\
& \left.-3\mathrm{e}^{-2\rho}\left(\partial_r^2\sigma + (\partial_r\sigma)^2\right)^2\right]
\end{aligned}
$$

$$
\begin{aligned}
&+r_0^2\left[-\frac{4}{3}b\mathrm{e}^{-2\rho}\left(\partial_r^2\rho\right)^2\ln\left(\frac{R_4}{\mu^2}\right)+\frac{4}{3}b\partial_r^2\left\{\mathrm{e}^{-2\rho}\partial_r^2\rho\ln\left(\frac{R_4}{\mu^2}\right)\right\}\right.\\
&+\frac{4b\mathrm{e}^{2\rho}}{3r_0^2}\ln\left(\frac{R_4}{\mu^2}\right)-\frac{4}{r_0^2}\left(\frac{b}{3}+b'\right)\partial_r^2\ln\left(\frac{R_4}{\mu^2}\right) \qquad (10)\\
&+\frac{4}{3}b\mathrm{e}^{-2\rho}\partial_r^2\rho\partial_r^2\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}-\frac{4}{3}b\partial_r^2\left\{\mathrm{e}^{-2\rho}\partial_r^2\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}\right\}\\
&+\frac{4\mathrm{e}^{-2\rho}\partial_r^2\rho}{R_4}\left\{\frac{2}{3}b\mathrm{e}^{-2\rho}\left(\partial_r^2\rho\right)^2+\frac{2b\mathrm{e}^{2\rho}}{3r_0^2}+\frac{1}{3}b\partial_r^2R_4-\frac{4}{r_0^2}\left(\frac{b}{3}+b'\right)\partial_r^2\rho\right\}\\
&\left.+\partial_r^2\left\{\frac{2\mathrm{e}^{-2\rho}}{R_4}\left\{\frac{2}{3}b\mathrm{e}^{-2\rho}(\partial_r^2\rho)^2+\frac{2b\mathrm{e}^{2\rho}}{3r_0^2}+\frac{1}{3}b\partial_r^2R_4-\frac{4}{r_0^2}\left(\frac{b}{3}+b'\right)\partial_r^2\rho\right\}\right\}\right]\ ,
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\;& -\frac{r_0^2}{16\pi G}\left[-2\mathrm{e}^{2\sigma}\left\{3\partial_r^2\sigma+3\left(\partial_r\sigma\right)^2+\partial_r^2\rho\right\}+\frac{4}{r_0^2}\mathrm{e}^{2\rho+2\sigma}\right]\\
&+b'r_0^2\left[2\partial_r^2(\mathrm{e}^{-2\rho}\partial_r^2\sigma)+\frac{4}{3}\left(\mathrm{e}^{-2\rho}\partial_r^2\rho\partial_r^2\sigma+\partial_r^2(\sigma\mathrm{e}^{-2\rho}\partial_r^2\rho)\right)\right.\\
&\left.-\frac{8}{3r_0^2}\partial_r^2\sigma-\frac{1}{6}\left\{\partial_rR_4\partial_r\sigma+\partial_rR_4\partial_r\sigma\right\}\right]\\
&-\left\{b''+\frac{2}{3}(b+b')\right\}r_0^2\left[-\frac{1}{2}\partial_r^2R_4+\frac{1}{2}\left\{\partial_rR_4\partial_r\sigma+\partial_rR_4\partial_r\sigma\right\}\right.\\
&\left.+3\partial_r^2\left\{\mathrm{e}^{-2\rho}\left(\partial_r^2\sigma+(\partial_r\sigma)^2\right)\right\}-6\partial_r\left\{\mathrm{e}^{-2\rho}\partial_r\sigma\left(\partial_r^2\sigma+(\partial_r\sigma)^2\right)\right\}\right] \qquad (11)
\end{aligned}
$$

Note that now the real 4 dimensional metric is given by

$$ds^2=-\mathrm{e}^{2\sigma+2\rho}dx^+dx^-+r_0^2\mathrm{e}^{2\sigma}d\Omega^2\ . \qquad (12)$$

Let us make the following change: ∂_r etc. as

$$\partial_r=\mathrm{e}^{\rho+\sigma}\partial_l\ ,\quad \partial_r^2=\mathrm{e}^{2\rho+2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\ . \qquad (13)$$

Using the new radial coordinate l, the metric (12) is rewritten as follows:

$$
\begin{aligned}
ds^2 &= -f(l)dt^2+dl^2+r(l)^2d\Omega^2\\
f(l) &= \mathrm{e}^{2\sigma+2\rho}\\
r(l) &= r_0\mathrm{e}^{\sigma}\ . \qquad (14)
\end{aligned}
$$

Here $f(l)$ is called a redshift function and $r(l)$ is a shape function. If $f(l)$ and $r(l)$ are smooth positive-definite functions, which satisfy the conditions:

$$
\begin{aligned}
&f(l)\to 1,\ r(l)\to l \quad \text{when } |l|\to\infty\\
&f(l),\ r(l)\to \text{ finite} \quad \text{when } |l|\to 0\ , \qquad (15)
\end{aligned}
$$

the metric expresses the wormhole which connects two asymptotically flat universes.

For the choice of the metric (14), Eqs.(8), (9), (10) and (11) are rewritten as follows:

$$0 = -\frac{r_0^2}{48\pi G}\mathrm{e}^{4\rho+4\sigma}\left[-\partial_l^2\sigma+\partial_l\sigma\partial_l\rho\right]$$

$$
\begin{aligned}
&+\frac{b' r_0^2 \mathrm{e}^{4\rho+4\sigma}}{16}\left[-8\mathrm{e}^{-2\sigma}\partial_l\sigma\partial_l\left(\mathrm{e}^{2\sigma}\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+(\partial_l\sigma)^2\right)\right)\right.\\
&+8\sigma\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+(\partial_l\sigma)^2\right)\left(\partial_l^2\rho+(\partial_l\rho)^2+\partial_l\sigma\partial_l\rho\right)\\
&\left.+\frac{8}{3}\mathrm{e}^{-2\sigma}\partial_l\sigma\partial_l\left\{R_4\sigma\right\}+\frac{32}{3}\mathrm{e}^{-2\sigma}\sigma\partial_l R_4\partial_l\sigma\right]\\
&-\left\{b''+\frac{2}{3}(b+b')\right\}\frac{r_0^2\mathrm{e}^{4\rho+4\sigma}}{16}\left[16\mathrm{e}^{-2\sigma}\partial_l\sigma\partial_l R_4+4(\partial_l\sigma)^2\left(\partial_l^2\rho+(\partial_l\rho)^2+\partial_l\sigma\partial_l\rho\right)\right.\\
&-12\mathrm{e}^{-2\sigma}\partial_l\sigma\partial_l\left(\mathrm{e}^{2\sigma}\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+2\left(\partial_l\sigma\right)^2\right)\right)\\
&\left.+12\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+2\left(\partial_l\sigma\right)^2\right)\left(\partial_l\sigma\right)^2\right]\\
&-\frac{r_0^2\mathrm{e}^{2\rho+2\sigma}}{16}\left\{\frac{1}{2}\partial_l^2\rho-\frac{3}{2}(\partial_l\rho)^2+\frac{1}{2}\partial_l\sigma\partial_l\rho-\frac{1}{4}\partial_l^2+\frac{5}{4}\partial_l\rho\partial_l-\frac{1}{4}\partial_l\sigma\partial_l\right\}\\
&\times\mathrm{e}^{2\rho+2\sigma}\left[\frac{16}{3}b'\left(-\sigma\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+(\partial_l\sigma)^2\right)+(\partial_l\sigma)^2\right)\right.\\
&\left.-\left\{b''+\frac{2}{3}(b+b')\right\}\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+2\left(\partial_l\sigma\right)^2\right)\right]\\
&+r_0^2\left[-\frac{1}{12}b\mathrm{e}^{4\rho+2\sigma}\partial_l\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}\partial_l R_4\right. \qquad (16)\\
&-\mathrm{e}^{2\rho+2\sigma}\left\{\frac{1}{2}\partial_l^2\rho-\frac{3}{2}(\partial_l\rho)^2+\frac{1}{2}\partial_l\sigma\partial_l\rho-\frac{1}{4}\partial_l^2+\frac{5}{4}\partial_l\rho\partial_l-\frac{1}{4}\partial_l\sigma\partial_l\right\}\\
&\times\mathrm{e}^{2\rho+2\sigma}\left\{\frac{2}{3}b\left(\partial_l^2\rho+(\partial_l\rho)^2+\partial_l\sigma\partial_l\rho\right)\ln\left(\frac{R_4}{\mu^2}\right)\right.\\
&+\frac{b}{4}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}\\
&+\left\{b\left(\frac{2}{3}\mathrm{e}^{-2\rho}\left(\partial_l^2\rho+(\partial_l\rho)^2+\partial_l\sigma\partial_l\rho\right)^2+\frac{2\mathrm{e}^{-2\sigma}}{3r_0^2}\right.\right.\\
&\left.+\frac{1}{3}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)R_4\right)\\
&\left.\left.\left.+\frac{4}{r_0^2}\left(\frac{b}{3}+b'\right)\left(\partial_l^2\rho+(\partial_l\rho)^2+\partial_l\sigma\partial_l\rho\right)\right\}\frac{1}{R_4}\right\}\right]\ ,\\
R_4 \equiv\ &-2\mathrm{e}^{-2\sigma}\left(\partial_l^2\rho+(\partial_l\rho)^2+\partial_l\sigma\partial_l\rho\right)+\frac{2}{r_0^2}\ , \qquad (17)\\
0 =\ &-\frac{r_0^2\mathrm{e}^{2\rho}}{16\pi G}\left[-\mathrm{e}^{2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\mathrm{e}^{2\sigma}+\frac{4}{r_0^2}\mathrm{e}^{2\sigma}\right]\\
&+b' r_0^2\mathrm{e}^{2\rho+4\sigma}\left\{-2\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+(\partial_l\sigma)^2\right)^2\right.\\
&-\frac{8}{3}\left(\partial_l^2\rho+(\partial_l\rho)^2+\partial_l\sigma\partial_l\rho\right)\left(\sigma\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+(\partial_l\sigma)^2\right)\right)\\
&+\frac{4}{3}\mathrm{e}^{-2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left(\sigma\mathrm{e}^{2\sigma}\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+(\partial_l\sigma)^2\right)\right)\\
&-\frac{1}{3}\left\{-2\left(\partial_l\sigma\right)^2\left(\partial_l^2\rho+(\partial_l\rho)^2+\partial_l\sigma\partial_l\rho\right)\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\mathrm{e}^{-2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left(\mathrm{e}^{2\sigma}\left(\partial_l\sigma\right)^2\right)\Big\}\Big\} \\
&-\left\{b''+\frac{2}{3}(b+b')\right\}r_0^2\mathrm{e}^{2\rho+4\sigma} \\
&\times\Big[\mathrm{e}^{-2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left(\mathrm{e}^{2\sigma}\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+2\left(\partial_l\sigma\right)^2\right)\right) \\
&-3\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+2\left(\partial_l\sigma\right)^2\right)^2\Big] \qquad (18)\\
&+r_0^2\mathrm{e}^{2\rho+4\sigma}\left[-\frac{4}{3}b\left(\partial_l^2\rho+\left(\partial_l\rho\right)^2+\partial_l\sigma\partial_l\rho\right)^2\ln\left(\frac{R_4}{\mu^2}\right)\right. \\
&+\frac{4}{3}b\mathrm{e}^{-2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left\{\mathrm{e}^{2\sigma}\left(\partial_l^2\rho+\left(\partial_l\rho\right)^2+\partial_l\sigma\partial_l\rho\right)\ln\left(\frac{R_4}{\mu^2}\right)\right\} \\
&+\frac{4b\mathrm{e}^{-4\sigma}}{3r_0^2}\ln\left(\frac{R_4}{\mu^2}\right)-\frac{4}{r_0^2}\left(\frac{b}{3}+b'\right)\mathrm{e}^{-2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\ln\left(\frac{R_4}{\mu^2}\right) \\
&+\frac{4}{3}b\left(\partial_l^2\rho+\left(\partial_l\rho\right)^2+\partial_l\sigma\partial_l\rho\right)\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\} \\
&-\frac{4}{3}b\mathrm{e}^{-2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left\{\mathrm{e}^{2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}\right\} \\
&+\frac{4\mathrm{e}^{2\sigma}\left(\partial_l^2\rho+\left(\partial_l\rho\right)^2+\partial_l\sigma\partial_l\rho\right)}{R_4}\left\{\frac{2}{3}b\mathrm{e}^{2\sigma}\left(\partial_l^2\rho+\left(\partial_l\rho\right)^2+\partial_l\sigma\partial_l\rho\right)^2\right. \\
&+\frac{2b\mathrm{e}^{-4\sigma}}{3r_0^2}+\frac{\mathrm{e}^{-2\sigma}}{3}b\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)R_4 \\
&\left.-\frac{4}{r_0^2}\left(\frac{b}{3}+b'\right)\mathrm{e}^{-2\sigma}\left(\partial_l^2\rho+\left(\partial_l\rho\right)^2+\partial_l\sigma\partial_l\rho\right)\rho\right\} \\
&+\mathrm{e}^{-2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left\{\frac{2}{R_4}\left\{\frac{2}{3}b\mathrm{e}^{4\sigma}\left(\partial_l^2\rho+\left(\partial_l\rho\right)^2+\partial_l\sigma\partial_l\rho\right)^2\right.\right. \\
&+\frac{2b}{3r_0^2}+\frac{1}{3}b\mathrm{e}^{2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)R_4 \\
&\left.\left.\left.-\frac{4\mathrm{e}^{2\sigma}}{r_0^2}\left(\frac{b}{3}+b'\right)\left(\partial_l^2\rho+\left(\partial_l\rho\right)^2+\partial_l\sigma\partial_l\rho\right)\right\}\right\}\right]\ ,
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\;& -\frac{r_0^2\mathrm{e}^{2\rho}}{16\pi G}\left[-2\mathrm{e}^{4\sigma}\left\{3\partial_l^2\sigma+4\partial_l\rho\partial_l\sigma+6\left(\partial_l\sigma\right)^2+\partial_l^2\rho+\left(\partial_l\rho\right)^2\right\}+\frac{4}{r_0^2}\mathrm{e}^{2\sigma}\right] \\
&+b'r_0^2\mathrm{e}^{2\rho}\Big[2\mathrm{e}^{2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left(\mathrm{e}^{2\sigma}\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+\left(\partial_l\sigma\right)^2\right)\right) \\
&+\frac{4}{3}\Big(\mathrm{e}^{4\sigma}\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+\left(\partial_l\sigma\right)^2\right)\left(\partial_l^2\rho+\left(\partial_l\rho\right)^2+\partial_l\sigma\partial_l\rho\right) \\
&+\mathrm{e}^{2\sigma}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left(\sigma\mathrm{e}^{2\sigma}\left(\partial_l^2\rho+\left(\partial_l\rho\right)^2+\partial_l\sigma\partial_l\rho\right)\rho\right)\Big) \\
&-\frac{8\mathrm{e}^{2\sigma}}{3r_0^2}\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+\left(\partial_l\sigma\right)^2\right)-\frac{\mathrm{e}^{2\sigma}}{3}\partial_lR_4\partial_l\sigma\Big] \\
&-\left\{b''+\frac{2}{3}(b+b')\right\}r_0^2\mathrm{e}^{2\rho+2\sigma}\Big[-\frac{1}{2}\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)R_4+\partial_lR_4\partial_l\sigma \qquad (19)\\
&+3\left(\partial_l^2+\partial_l\rho\partial_l+\partial_l\sigma\partial_l\right)\left\{\mathrm{e}^{2\sigma}\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+2\left(\partial_l\sigma\right)^2\right)\right\}
\end{aligned}
$$

$$-6\mathrm{e}^{-\sigma-\rho}\partial_l\left\{\mathrm{e}^{\rho+3\sigma}\partial_l\sigma\left(\partial_l^2\sigma+\partial_l\rho\partial_l\sigma+2\left(\partial_l\sigma\right)^2\right)\right\}\Big] \ .$$

The equations (16) and (18) contain the 6th order derivatives of ρ, $\partial_l^6\rho$ and the equations (16), (18) and (19) contain the 4th order derivatives of σ. Therefore we need to impose the initial conditions including 5th order derivative of ρ and 3rd order derivative of σ:

$$\begin{aligned}&\partial_l\rho|_{l=0}=\partial_l^2\rho|_{l=0}=\partial_l^3\rho|_{l=0}=\partial_l^4\rho|_{l=0}=\partial_l^5\rho|_{l=0}=0\\&\partial_l\sigma|_{l=0}=\partial_l^2\sigma|_{l=0}=\partial_l^3\sigma|_{l=0}=0 \ .\end{aligned}\tag{20}$$

The equations (16) and (18) contain the 5th order derivatives of σ from $\partial_l^4 R_4$ but $\partial_l^5\sigma$ always appears in the form $\partial_l^5\sigma\partial\rho$. Therefore these terms vanish at $l=0$. This tells that we cannot impose the initial condition $\partial_l^4\sigma=0$ since we cannot solve the differential equations with respect to $\partial_l^5\sigma$ at $l=0$.

We should also note that all the equations (16), (18) and (19) are not the dynamical equations of motion but the combinations with the constraint. The constraint can be found by putting $l=0$ under the initial conditions (20), when the equations (16) and (18) have the following form:

$$0 = \left(b+b'+\frac{3}{2}b''+8b'\sigma_0\right)\sigma_0^{(4)}+14b\mathrm{e}^{-2\sigma_0}{r_0}^2\rho_0^{(6)} \ , \tag{21}$$

$$\begin{aligned}0 &= \frac{3\mathrm{e}^{-2\sigma_0}}{8\,G\,\pi}-2b\mathrm{e}^{-4\sigma_0}\ln\left(\frac{2}{\mu^2{r_0}^2}\right)\\&\quad+{r_0}^2\left(b+b'+\frac{3}{2}b''-2b'\sigma_0\right)\sigma_0^{(4)}-b\mathrm{e}^{-2\sigma_0}{r_0}^4\rho_0^{(6)} \ ,\end{aligned}\tag{22}$$

$$0 = \frac{\mathrm{e}^{-2\,\sigma_0}}{G\,\pi\,{r_0}^2}+4\left(2\,b+3\,b''\right)\sigma_0^{(4)} \ . \tag{23}$$

Equations (21), (22) and (23) will be compatible if σ_0 satisfies to the following equation:

$$\begin{aligned}0 &= -224\,b\,\left(2\,b+3\,b''\right)\,G\,\pi\,\ln\left(\frac{2}{\mu^2\,{r_0}^2}\right)\\&\quad+\mathrm{e}^{2\,\sigma_0}\,\left(54\,b-30\,b'+81\,b''+40\,b'\,\sigma_0\right) \ .\end{aligned}\tag{24}$$

Then we can choose the initial conditions as (20) and (24) with arbitrary $\rho|_{l=0}$. Then under these initial conditions, we can use any two of equations (16), (18) and (19) as dynamical equations of motion. Equation (24) admits the real solution for σ_0 if

$$b\,\left(112\,b+168\,b''\right)\,\mathrm{e}^{-\frac{1}{2}+\frac{27\,(2\,b+3\,b'')}{20\,b'}}\,G\,\pi\,\ln\left(\frac{2}{\mu^2\,{r_0}^2}\right) < -5\,b' \tag{25}$$

and from (24) we have the restriction

$$\sigma_0 \geq \frac{1}{4}+\frac{54\,b+81\,b''}{40\,|b'|} \tag{26}$$

From equations (16), (18), (19) and initial conditions (20) we get the following behavior of ρ and σ near $l=0$

$$\rho(l) = \rho_0+\frac{2b+2b'+3b''+16b'\sigma_0}{80640\,G\,\pi\,{r_0}^4\,b\,(2b+3b'')}\,l^6+O(l^8) \tag{27}$$

$$\sigma(l) = \sigma_0-\frac{\mathrm{e}^{-2\sigma_0}}{96\,G\,\pi\,{r_0}^2\,(2b+3b'')}\,l^4+O(l^6) \tag{28}$$

Then for initial conditions (20) $\sigma(l)$ and $\rho(l)$ are decreasing functions near $l = 0$.

For the numerical calculation as an example we will consider $\mathcal{N} = 4$ $SU(N)$ super-YM theory where (see, for example, [10])

$$b = -b' = \frac{N^2 - 1}{(8\,\pi)^2}, \qquad b'' = 0. \tag{29}$$

Such theory became popular recently in relation with AdS/CFT correspondence. We take it as typical example of GUT. Note also that as it was mentioned in ref.[10] the explicit choice of b'' does not influence the equations of motion.

Then from (27-28) we get

$$\rho(l) \;=\; \rho_0 + \frac{-\sigma_0}{10080\,G\,\pi\,{r_0}^4\,b}\,l^6 + O(l^8), \tag{30}$$

$$\sigma(l) \;=\; \sigma_0 - \frac{\mathrm{e}^{-2\sigma_0}}{198\,G\,\pi\,{r_0}^2\,b}\,l^4 + O(l^6) \tag{31}$$

and from (24-25) we obtain

$$(\mu r_0)^2 > 2 \;\;\rightarrow\;\; b > 0, \quad \sigma_0 > 21/10 \tag{32}$$

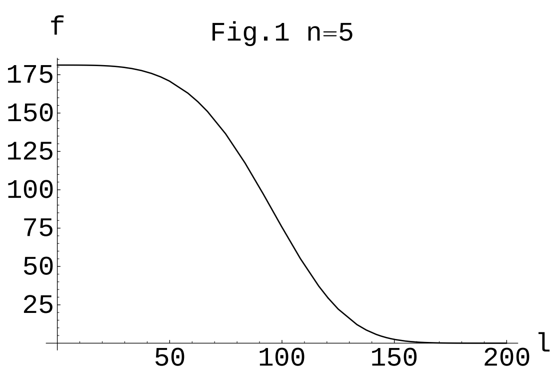
$$(\mu r_0)^2 = 2 \;\;\rightarrow\;\; b > 0, \quad \sigma_0 = 21/10 \tag{33}$$

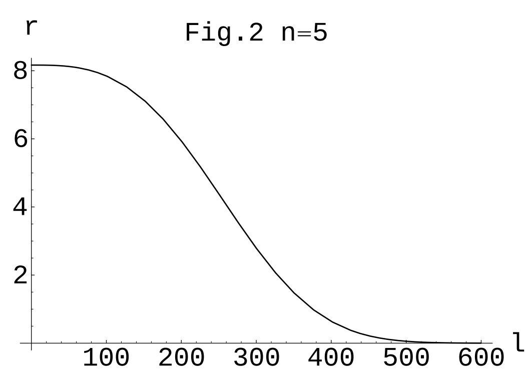
$$(\mu r_0)^2 < 2 \;\;\rightarrow\;\; 0 < b < \frac{5\,\mathrm{e}^{16/5}}{112\,G\,\pi\,\ln(\frac{2}{\mu^2\,{r_0}^2})}, \quad \frac{8}{5} < \sigma_0 < \frac{21}{10}. \tag{34}$$

Let us choose for numeric investigation, that

$$G = 1, \quad \mu^2 = 2, \quad {r_0}^2 = 2/\mu^2 = 1.$$

The results of numerical calculation are given by Figures 1 and 2 for redshift and shape functions where $N = 5$.





Hence we showed that for above choice of initial conditions the inducing of primordial wormholes at the early Universe is rather unrealistic. The throat radius of wormhole quickly shrinks to zero.

Let us now consider another choice of initial conditions

$$\partial_l\rho|_{l=0} = \partial_l^2\rho|_{l=0} = \partial_l^3\rho|_{l=0} = 0, \quad \partial_l^4\rho|_{l=0} \neq 0$$
$$\partial_l\sigma|_{l=0} = \partial_l^2\sigma|_{l=0} = \partial_l^3\sigma|_{l=0} = 0\;. \tag{35}$$

From equations (16), (18), (19) and initial conditions (35) we get the following behavior of ρ and σ near $l=0$

$$\partial_l^4\sigma|_{l=0} = \frac{1}{792b^2G\pi}\Big[-99b\mathrm{e}^{-2\sigma_0} + 4b^2G\pi\rho_0\sigma_0\left(-51+36\mathrm{e}^{4\sigma_0}+20\mathrm{e}^{4\sigma_0}\rho_0{\sigma_0}^2\right)$$

$$-4\varepsilon\sqrt{\pi}\rho_0\sigma_0\sqrt{b^3G\left(99\mathrm{e}^{2\sigma_0}(21-10\sigma_0)+bG\pi(-51+36\mathrm{e}^{4\sigma_0}+20\mathrm{e}^{4\sigma_0}\rho_0{\sigma_0}^2)^2\right)}\Big] \tag{36}$$

$$\partial_l^4\rho|_{l=0} = \frac{1}{132\,b^2\,G\,\sqrt{\pi}}\Big[b^2\,G\,\sqrt{\pi}\,\left(51-36\,\mathrm{e}^{4\,\sigma_0}-20\,\mathrm{e}^{4\,\sigma_0}\,\rho_0\,{\sigma_0}^2\right)$$

$$+\,\varepsilon\sqrt{b^3\,G\,\left(99\,\mathrm{e}^{2\,\sigma_0}\,(21-10\,\sigma_0)+b\,G\,\pi\,(-51+36\,\mathrm{e}^{4\,\sigma_0}+20\,\mathrm{e}^{4\,\sigma_0}\,\rho_0\,{\sigma_0}^2)^2\right)}\Big] \tag{37}$$

where $\varepsilon=\pm1$.

Moreover we obtain the conditions

$$\partial_l^5\rho|_{l=0} = \partial_l^5\sigma|_{l=0} = 0, \tag{38}$$

or

$$891\,\mathrm{e}^{2\,\sigma_0}\,(10\,\sigma_0-21) = 8\,b\,G\,\pi\left(-51+36\,\mathrm{e}^{4\,\sigma_0}+20\,\mathrm{e}^{4\,\sigma_0}\,\rho_0\,{\sigma_0}^2\right)^2, \quad \sigma_0 > \frac{21}{10}. \tag{39}$$

For case $\sigma_0 = 21/10$ we have

$$\partial_l^4\sigma|_{l=0} = \frac{1}{2200}\left(\frac{-275\,\mathrm{e}^{-21/5}}{b\,G\,\pi}+(1680\,\mathrm{e}^{42/5}-2380)\,\rho_0+4116\,\mathrm{e}^{42/5}\,{\rho_0}^2\right), \tag{40}$$

$$\partial_l^4\rho|_{l=0} = \frac{1}{110}\left(85-60\,\mathrm{e}^{42/5}-147\,\mathrm{e}^{42/5}\,\rho_0\right) \tag{41}$$

Then functions $\sigma(l)$ and $\rho(l)$ will increase near $l=0$ if

$$\rho_0 < \frac{5}{294\,b\,G}\left[b\,G\left(-12+17\,\mathrm{e}^{-42/5}\right)-\frac{\mathrm{e}^{-42/5}}{\sqrt{\pi}}\sqrt{b\,G\left(231\,\mathrm{e}^{\frac{21}{5}}+b\left(17-12\,\mathrm{e}^{\frac{42}{5}}\right)^2G\,\pi\right)}\right]$$

for $G=1$ and $b=(N^2-1)/(8\pi)^2$ we have $\rho_0 < -0.4081$. The explicit numerical study (see Figures 3 and 4) shows that shape and redshift functions are increasing giving the window for inducing of primordial wormholes.

Fig.3 n=10

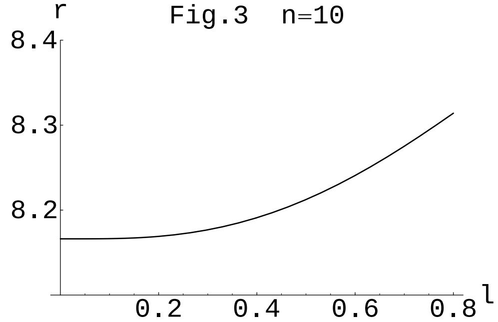
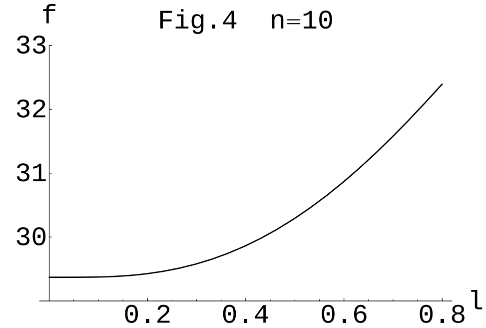


Fig.4 n=10



On these figures $\sigma_0 = 21/10$ and $\rho_0 = -0.41$.

Hence we numerically proved that at least for some initial conditions GUTs at the early Universe may help in producing of primordial wormholes. It is of course the open question which initial conditions from the two classes presented above lead to more stable configuration. Moreover, due to complicated structure of field equations and initial conditions itself we cannot classify from the very beginning the initial conditions as supporting (or not) wormholes production. Nevertheless for any specific GUT under discussion above study may be easily repeated and principal possibility of wormholes inducing may be shown at least numerically.

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